D1 SIS IAL

Write your answers in the D1 answer book for this paper.

1.

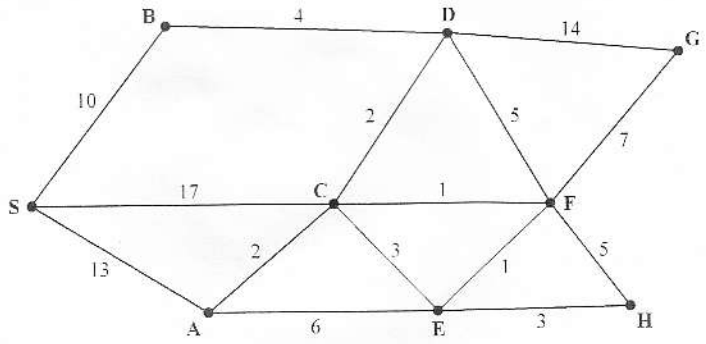


Figure 1

Figure 1 represents a network of roads. The number on each arc gives the length, in km, of the corresponding road.

(a) Use Dijkstra's algorithm to find the shortest distance from S to G. State the shortest route.

(6)

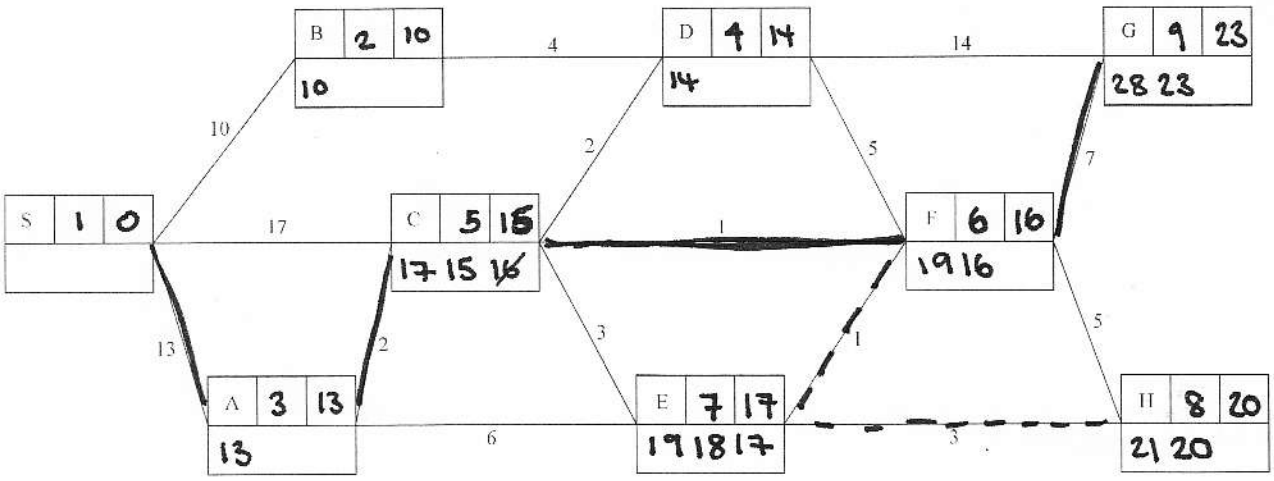
(b) State both the shortest distance and the shortest route from S to H.

(2)

(Total 8 marks)

S15 IAL D1

1.



SACFG (23)

SACFEH (20)

Key:

Vertex	Order of labelling	Final value
Working values		

2. A list of n numbers needs to be sorted into **descending** order starting at the left-hand end of the list.

(a) **Describe** how to carry out the first pass of a **bubble** sort on the numbers in the list.

(2)

(b) (i) State which number in the list is guaranteed to be in the correct final position after the first pass.

(ii) Write down the maximum number of passes required to sort a list of n numbers.

(2)

(c) The numbers below are to be sorted into **descending** order. Use a bubble sort, starting at the left-hand end of the list, to obtain the sorted list. You need only give the state of the list after each pass.

11 9 4 13 5 1 7 12 8

(4)

(d) Apply the first-fit decreasing bin packing algorithm to your ordered list to pack the numbers into bins of size 21

(2)

(Total 10 marks)

a) Compare pairs of numbers from left to right, swapping if the number on the left is smaller than that to its right

b) Smallest

ii) $n-1$ passes
1 final to check.

c) 11 9 4 13 5 1 7 12 8

11 9 13 5 4 7 12 8 1 (1)

11 13 9 5 7 12 8 4 1 (2)

13 11 9 7 12 8 5 4 1 (3)

13 11 9 12 8 7 5 4 1 (4)

13 11 12 9 8 7 5 4 1 (5)

13 12 11 9 8 7 5 4 1 (6)

13 12 11 9 8 7 5 4 1 (7)

d) Bin1: 13 8 —

Bin2: 12 9 —

Bin3: 11 7 1 — (2)

Bin4: 5 4 — (12)

14 wasted

3.

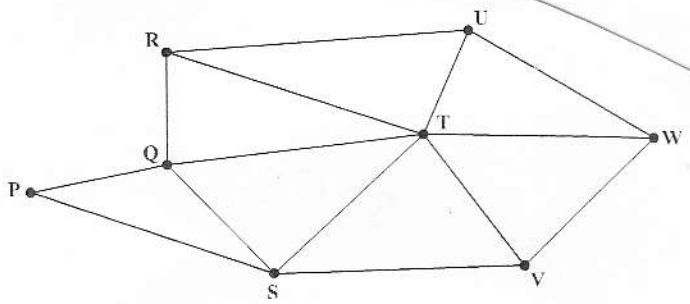


Figure 2

Figure 2 shows a graph G.

(a) Write down an example of a cycle on G.

(1)

(b) State, with a reason, whether or not P – Q – R – T – Q – S is an example of a path on G.

(2)

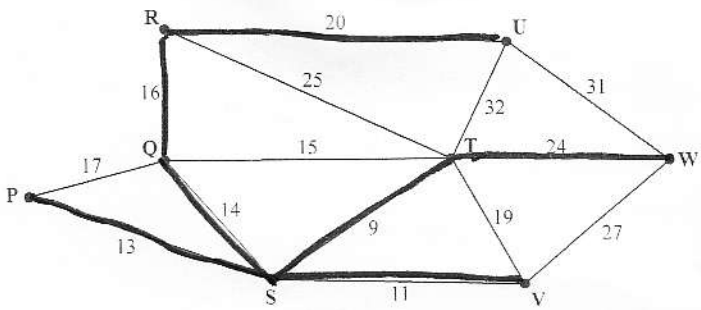


Figure 3

The numbers on the 14 arcs in Figure 3 represent the distances, in km, between eight vertices, P, Q, R, S, T, U, V and W, in a network.

a) R-T-Q-R

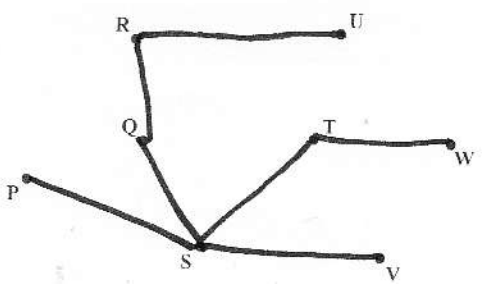
b) not a path as a node is repeated

c) PS; ST; SV; SQ; QR; RU; TW (107)

d) ST; SV; SP; QS; QTx; QR, QPx; TUx
RU; TW reject all other (107)

f) 30

(e)



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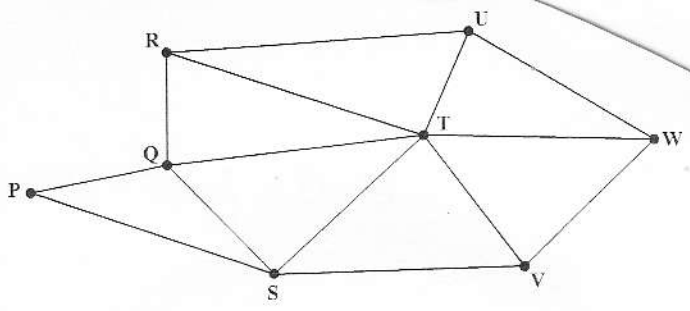

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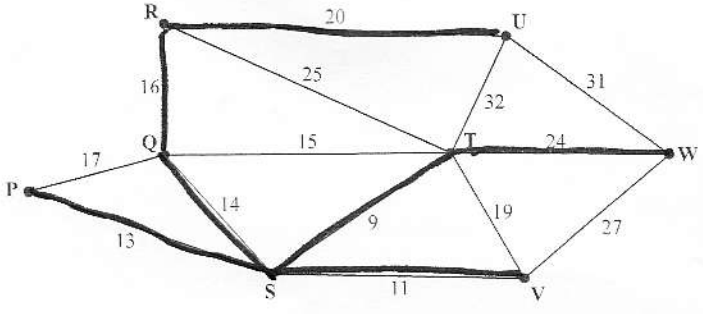


Figure 3

The numbers on the 14 arcs in Figure 3 represent the distances, in km, between eight vertices, P, Q, R, S, T, U, V and W, in a network.

(c) Use **Prim's** algorithm, starting at P, to find the minimum spanning tree for the network. You must clearly state the order in which you select the arcs of your tree.

(3)

(d) Use **Kruskal's** algorithm to find the minimum spanning tree for the network. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to the minimum spanning tree.

(3)

(e) Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book.

(1)

The weight on arc RU is now increased to a value of x. The minimum spanning tree for the network is still unique and includes the same arcs as those found in (e).

(f) Write down the smallest interval that must contain x.

(2)

(Total 12 marks)

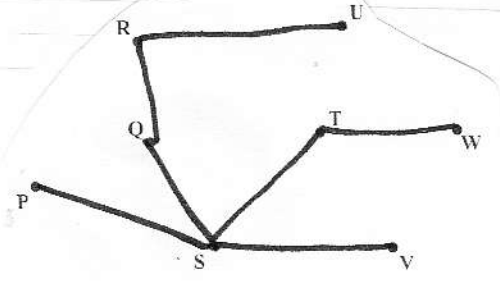
a) R-T-Q-R

b) not a path as a node is repeated

c) PS; ST; SV; SQ; QR; RU; TW (107)

d) ST; SV; SP; QS; QTx; QR, QPx; TVx
RU; TW reject all other (107)

f) 30



4. (a) Define the term 'alternating path'.

(2)

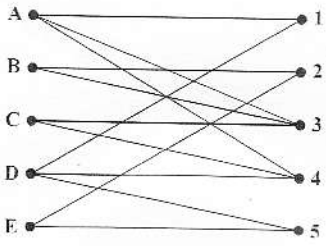


Figure 4

Figure 4 shows the possible allocations of five people, A, B, C, D and E, to five tasks, 1, 2, 3, 4 and 5

An initial matching has three people allocated to three of the tasks.

Starting from this initial matching, one possible alternating path that starts at E is

E – 2 – B – 3 – A – 4 – D – 1

(b) Use this information to

(i) deduce this initial matching.

(ii) list the improved matching generated by the given alternating path.

(2)

(c) Starting from the improved matching found in (b), use the maximum matching algorithm to obtain a complete matching. You must list the alternating path you use and the final matching.

(3)

(Total 7 marks)

4. (a)

A path from an unmatched vertex in set X to an unmatched vertex in Y, which alternates between arcs not in the matching with arcs in the matching.

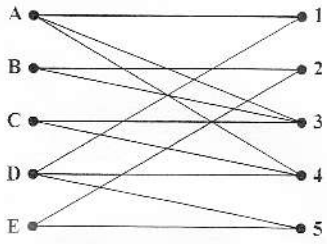


Figure 4

b) i) B = 2
A = 3
D = 4
C, E – unmatched.

ii) E = 2
B = 3
A = 4
D = 1
C – unmatched.

c) C – 4 = A – 1 = D – 5

OR C = 4 – A = 1 – D = 5

Complete
A = 1
B = 3
C = 4
D = 5
E = 2

5.

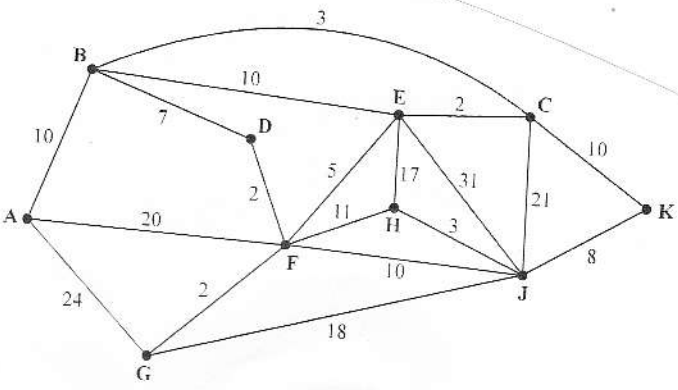


Figure 5

[The total weight of the network is 214]

Figure 5 models a network of canals that have to be inspected. The number on each arc represents the length, in km, of the corresponding canal. Priya needs to travel along each canal at least once and wishes to minimise the length of her inspection route.

She must start and finish at A.

(a) Use the route inspection algorithm to find the length of her route. State the arcs that will need to be traversed twice. You should make your method and working clear.

(6)

(b) State the number of times that vertex F would appear in Priya's route.

(1)

It is now decided to start the inspection route at H. The route must still travel along each canal at least once but may finish at any vertex.

(c) Determine the finishing point so that the length of the route is minimised. You must give reasons for your answer and state the length of the minimum route.

(3)

(Total 10 marks)

AE 15 (BC) GH 13 (F) ⇒ 28 +

AG 21 (BDF) EH 16 (F) ⇒ 37

AH 30 (BDF) EG 7 (F) ⇒ 37

repeat AB; BC; CE; GF; FH + 28

b) 3

c) repeat EF, FG (+7) end at A

length = 221

Start/finish at different end points

∴ Semi-Eulerian solution is possible.

[The sum of the durations of all the activities is 142 days]

6

A project is modelled by the activity network shown in Figure 6. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Complete the precedence table in the answer book.

(2)

(b) Complete Diagram 1 in the answer book to show the early event times and late event times.

(4)

(c) Calculate the total float for activity D. You must make the numbers you use in your calculation clear.

(1)

(d) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

(1)

Diagram 2 in the answer book shows a partly completed scheduling diagram for this project.

(e) Complete the scheduling diagram, using the minimum number of workers, so that the project is completed in the minimum time.

(4)

(Total 12 marks)

6. (a)

Activity	Immediately preceding activities
A	-
B	-
C	-
D	A
E	B, C
F	B, C

Activity	Immediately preceding activities
G	C
H	C
I	D, E
J	G
K	G
L	H, G

Activity	Immediately preceding activities
M	I
N	D, E, F, J
P	D, E, F, J
Q	K, P
R	K, P

(b)

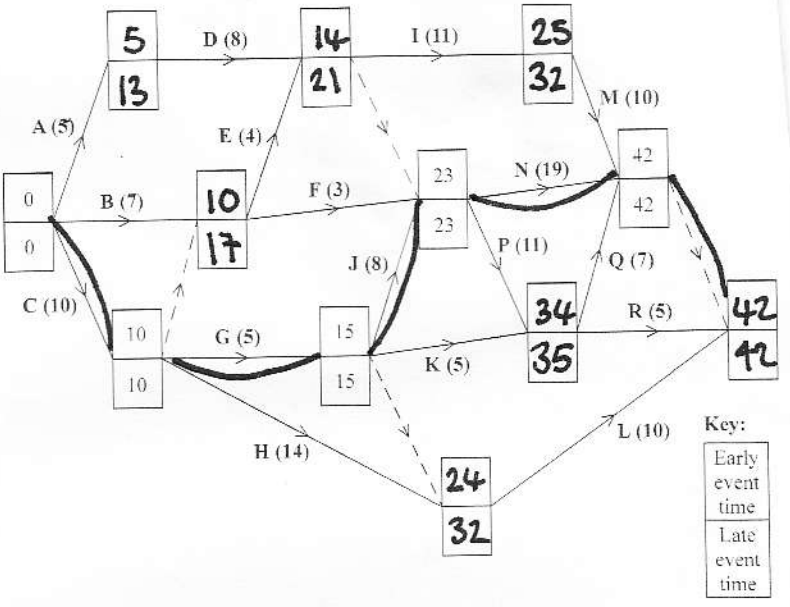
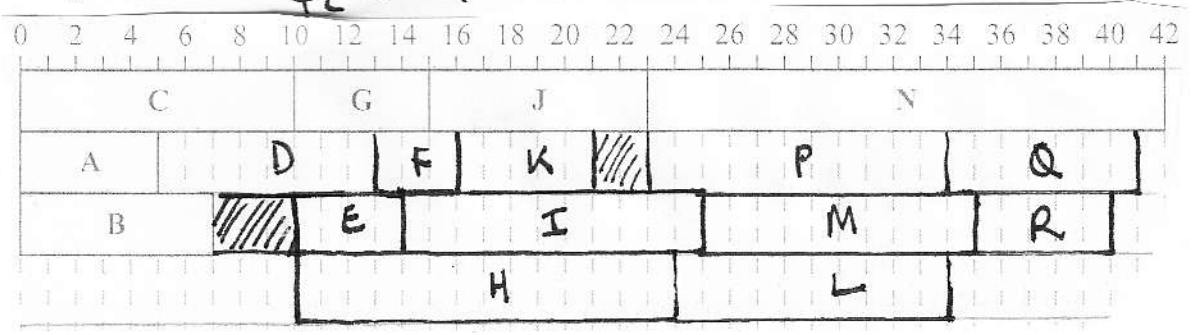


Diagram 1

c) float D $21 - 8 - 5 = 8$

d) LB = $\frac{142}{42} = 3\frac{4}{42} \rightarrow 4$ workers.



7. Ian plans to produce two types of book, hardbacks and paperbacks. He will use linear programming to determine the number of each type of book he should produce.

Let x represent the number of hardbacks Ian will produce.

Let y represent the number of paperbacks Ian will produce.

Each hardback takes 1 hour to print and 15 minutes to bind.

Each paperback takes 35 minutes to print and 24 minutes to bind.

The printing machine must be used for at least 14 hours. The binding machine must be used for at most 8 hours.

(a) (i) Show that the printing time restriction leads to the constraint $12x + 7y \geqslant k$, where k is a constant to be determined.

(ii) Write the binding time restriction in a similar simplified form.

(4)

Ian decides to produce at most twice as many hardbacks as paperbacks.

(b) Write down an inequality to model this constraint in terms of x and y.

(2)

(c) Add lines and shading to Diagram 1 in the answer book to represent the constraints found in (a) and (b). Hence determine, and label, the feasible region R.

(4)

Ian wishes to maximise $P = 60x + 36y$, where P is the total profit in pounds.

(d) (i) Use the objective line (ruler) method to find the optimal vertex, V, of the feasible region. You must draw and clearly label your objective line and the vertex V.

(ii) Determine the exact coordinates of V. You must show your working.

(4)

(e) Given that P is Ian's expected total profit, in pounds, find the number of each type of book that he should produce and his maximum expected profit.

(2)

(Total 16 marks)

7.

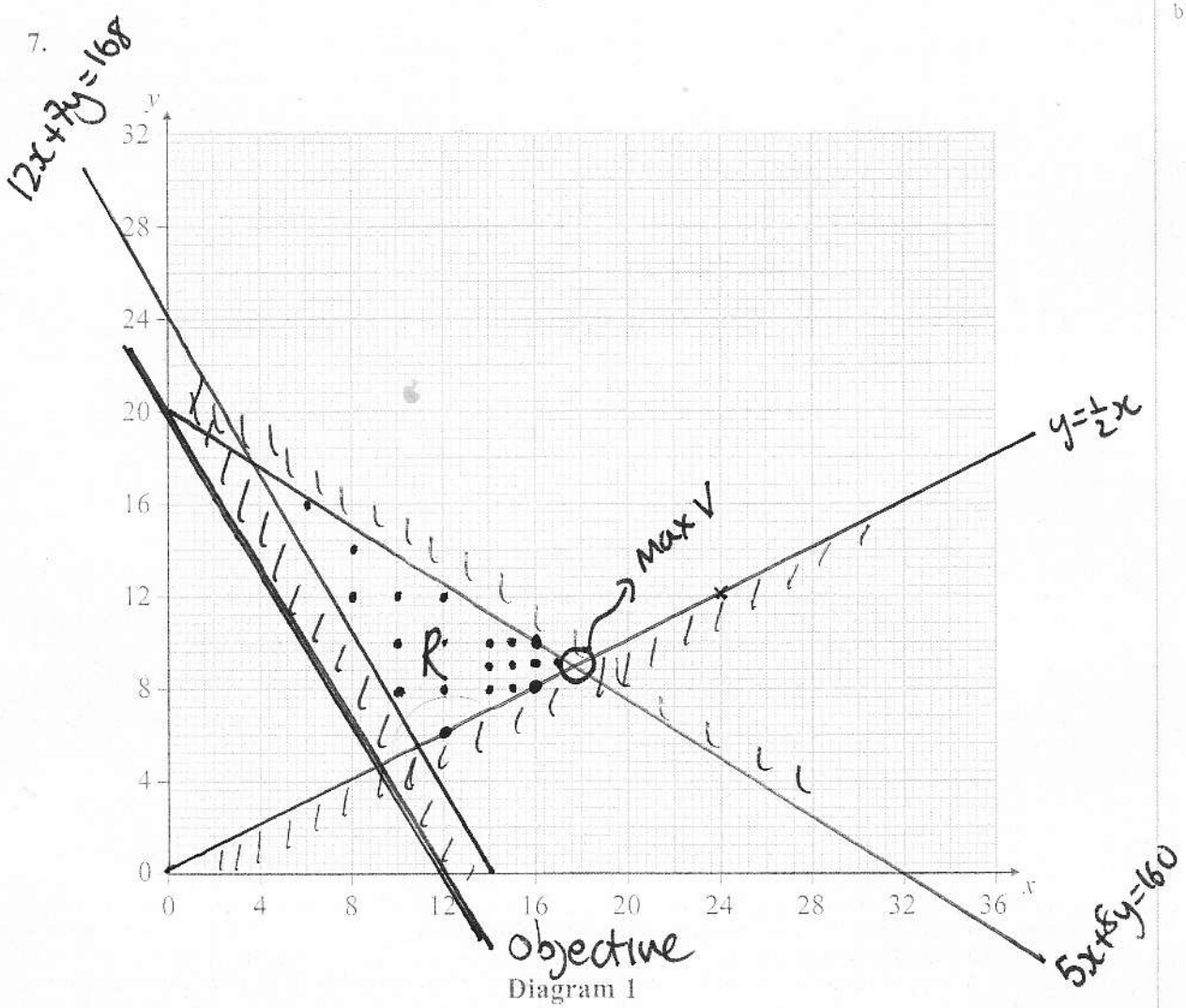


Diagram 1

i) $x + \frac{7}{12}y \geq 14$ ii) $\frac{1}{4}x + \frac{2}{5}y \leq 8$

(×12) $12x + 7y \geq 168$ (×20) $5x + 8y \leq 160$

$k = 168$

b) $x = 50$ $\therefore 2y \geq x$ $y \geq \frac{1}{2}x$

$y = 25$ (×2)

c) $P = 60x + 35y$ $(= 180)$ $x = 3$ $y = 5$

(×4) $x = 12$ $y = 20$

f) (17, 9)

17 hard backs

9 paperbacks

$P = £1344$

e) $5x + 8y = 160$ $y = \frac{1}{2}x \Rightarrow 8y = 4x$

$9x = 160$

$x = \frac{160}{9}$ $y = \frac{80}{9}$ $V\left(\frac{160}{9}, \frac{80}{9}\right)$